PEDAGOGICAL CONCEPTS AS GOALS FOR TEACHER EDUCATION: TOWARDS AN AGENDA FOR RESEARCH IN TEACHER DEVELOPMENT

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What conceptions of mathematics learning and teaching might contribute to increased mathematics teacher effectiveness? I argue that identification of goals for mathematics teacher education is critical to both effective teacher education and productive research on teacher development. Based on empirical and theoretical work in the context of three major research projects, I propose a set of pedagogical concepts for consideration as goals for teacher education. These concepts are proposed both because they are important to mathematics teaching and because they are generally not part of the pedagogical understanding of teachers in the US that we have studied. Successful fostering of these pedagogical concepts through teacher education will depend on research investigating teacher development of these concepts.

Underlying Assumptions

The arguments advanced in this theoretical article derive from both a social and a cognitive perspective. The use of these perspectives is pragmatic rather than the result of epistemological commitments. In addition, the arguments are based on two assumptions: 1. Aspects of the knowledge base in mathematics education are critical content (goals) for mathematics teacher education (1). 2. Identification of goals for mathematics teacher education is critical to both effective teacher education and productive research on teacher education.

Teacher education efforts, including those that are the context for research on teacher education, can be sorted into two categories: those with process goals only and those that have content and process goals. Highlighting the former category are programs that derive from the Japanese lesson study model (e.g., Yoshida, 1999)) and programs focused on teacher inquiry or teacher research (e.g., Dana, & Yendol-Silva, 2003). The basis of these programs is that the engagement of teachers in inquiry-based, reflective practices combined with appropriate support and communication structures can enable the ongoing professional development of mathematics teachers. These programs, which have demonstrated significant ongoing benefit for teachers of mathematics, are not focused on the learning of particular pedagogical principles (other than learning of the inquiry and communication processes).

The second category of teacher education efforts involves courses for teachers in which teacher educators plan for teacher learning of particular mathematics education concepts, skills, and dispositions. Although teacher education courses are often criticized as removed from practice and unresponsive to the needs and interests of the teachers involved, these negatives are not inherent properties of such an approach.

An assumption underlying this article (#1 above) is that there are understandings of mathematics learning and teaching that are important for teachers to develop. Therefore, although lesson study and teacher inquiry are important and useful, they are not sufficient. Courses that are designed to promote powerful ideas about learning and teaching are needed as well. This assumption (in conjunction with assumption #2 above) leads to the question, What pedagogical understandings would be useful foci for mathematics teacher education?

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In this article, I focus on teacher education that aims to promote teacher learning of particular aspects of the knowledge base on mathematics teaching; I discuss potential goals for teacher education of this type.

Current Articulation of Goals for Teacher Learning

Currently, the identification of goals for teacher education courses is largely a part of teacher educators' practices and not the focus of theoretical and empirical reports. A perusal of articles in the Journal of Mathematics Teacher Education since its inception shows a scarcity of discourse on this subject. Some articles focus on process goals such as developing reflective practitioners (e.g., McDuffie, 2004). Hiebert, Morris, and Glass (2003) focused on learning to teach from practice. Within this broad objective, they identified specific requisite dispositions and skills.

Literature that focuses more on specific learning includes reports of fostering teachers' understanding of students thinking (e.g., Crespo, 2000). Schifter, Bastable, and Russell (e.g., 1999) developed materials targeted at developing knowledge of students thinking as they learn particular mathematics and teacher reflection on related teacher interventions. The Cognitively Guided Instruction Project (Carpenter, Fennema, Franke, Levi, & Empson, 1999) focused on providing research-based information on students' solution strategies to teachers.

The Current State of the Knowledge Base in Mathematics Teaching

Many countries of the world have been involved in a reform in mathematics education over the last 15-20 years. The formal start of the reform in the United States is recognized to be the publication of the Standards (National Council Teachers of Mathematics, 1989). The reform has generally been an effort to focus mathematics instruction on conceptual learning, mathematical thinking, communication, and problem solving for all students. These goals for instruction have led to a decreased acceptance of direct instruction (teacher telling and showing) as the primary mode of teaching. Mathematics educators have replaced direct instruction with a set of reform strategies, such as the use of collaborative group problem solving, whole class discussions, manipulatives, software environments, calculators, and probing questions. Lacking are models of teaching -- frameworks for guiding the fostering of students' mathematical conceptions. As a result, teachers' use of reform curricula and strategies is often unprincipled and ineffective.

In many locales, there is neither a consensus model of teaching, nor a recognized set of alternative models. Rather teaching is implicitly defined by the curricula, the reform strategies, and the consensus "don'ts" (e.g., teacher telling, showing, giving answers) (2). The lack of clearly articulated, established models of teaching handicaps teacher education and research on teacher education. Without such models the goals of teacher education are at best under-defined. Teacher education tends to be directed towards broad skills (asking probing questions, focusing on students' thinking, writing lesson plans) as opposed to the development of particular pedagogical principles. In the next section, I identify potential goals for teacher education based on our emerging framework on mathematics learning and teaching.

Identifying Key Conceptions for Mathematics Teaching

Through three major research projects on teacher development grounded in the research literature, my colleagues and I have identified pedagogical concepts that seem to be important for high-quality mathematics teaching. This work has been interwoven with theoretical work on mathematics conceptual learning and teaching (Simon, Tzur, Heinz & Kinzel, 2004; Simon & Tzur, 2004; Tzur, & Simon, 2004). The pedagogical concepts that we have identified derive from the perspectives represented by this theoretical work. In this article, I identify key pedagogical concepts that derived from our work and the work of others in order to discuss goals for teacher education and agendas for research on teacher development. I make no attempt to provide an exhaustive list of concepts; rather I raise a subset for consideration.

Briefly, our theoretical work involves both social and cognitive perspectives. We use social perspectives to account for the norms that are negotiated in the classroom (McNeal & Simon, 2000) that afford and constrain the learning and communication in the classroom. We use a cognitive perspective to describe how new knowledge is developed from extant knowledge, particularly Piaget's constructs of assimilation and reflective abstraction.

Following are brief discussions of a set of five pedagogical concepts that are important to consider because of their impact on mathematics teaching and because we have found them to be generally lacking among mathematics teachers in US classrooms. Most of these concepts are overlapping and interrelated. Each of these concepts deserves extensive discussion. In lieu of space in this short article, the reader is referred to articles related to each of the concepts.

- Negotiation of classroom norms. The notion that classroom norms are negotiated, not imposed (McNeal & Simon, 2000; Yackel & Cobb, 1996), allows teachers to be conscious of their contribution to the constitution of classroom norms. This understanding of their role allows teachers to engage intentionally in the negotiation of norms that support rich mathematical classroom learning. Although mathematics researchers introduced the construct ten years ago, it has generally not been an explicit goal for teacher education.
- Assimilation. An understanding of assimilation is essential for teachers to understand the determinants of what students perceive and understand and to focus on the resources students bring to learning situations. Cobb, Yackel, & Wood (1992) described a representational view of mind to characterize educators' lack of understanding of assimilation. Our study of teachers involved in the reform (Simon, Tzur, Heinz, Kinzel, & Smith, 2000) highlighted the distinction between teachers with perception-based perspectives (lacking a concept of assimilation) and those with conception-based perspectives. Understanding assimilation affords better anticipation of student responses to lessons and teacher reflection as to why lessons were unsuccessful. It allows teachers to question assumptions that students' perceptions/experience are the same as the teachers'.
- What it means to develop a new mathematical operation. Teachers struggle with what it means for students to develop a new operation, for example multiplication. Teachers tend to teach about multiplication to students who have no concept of multiplication to learn about. Missing is the idea that the term "multiplication" must label for the student a commonality (abstraction) that they perceive in their actions in particular situations. It is only when students observe that what I did in this problem about the cost of 5 candy bars is "the same" as what I did in this problem about 7 boxes of pencils, that they have something to label as multiplication – that commonality. This perception of commonality builds on the learner's anticipation of the activity needed and the effect of that activity. This pedagogical concept is based on the concept of assimilation and the concept of learning through activity discussed next..

- Learning through activity. Teachers often focus on the dialogic aspects of teaching. Classroom discussions and small group conversations can be an important part of the learning process. However, teachers need to be able to do more than encourage participation in discussions of mathematical problems. It is helpful if teachers can think about how learners learn through their own goal-directed activity (Simon, et al, 2004; von Glasersfeld, 1995). Students' goals influence what they attend to. Their activity and reflection afford them a way to extend current conceptions and create new ones. Teacher understanding of learning through activity can contribute to effective selection, sequencing, and modification of mathematical tasks.
- Reflective abstraction versus empirical learning. Learning of mathematical concepts is not an empirical learning process (Simon, in press; 2003); rather it is a result of reflective abstraction. An empirical learning process is an inductive process through which learners discover patterns by observing a set of inputs and related outputs. Through an empirical process, learners learn that a pattern exists. The phenomenon that underlies the pattern remains a black box to the learner. Reflective abstraction, according to Piaget (2001), is the process by which higher-level mental structures are developed from lower-level structures, a coordination of actions leading to a new conception. He described it as having two phases, a projection phase in which the actions at one level become the objects of reflection at the next and a reflection phase in which a reorganization takes place. Reflective abstraction develops anticipation of the logical necessity of a mathematical relationship. Teacher awareness of this distinction helps them make students' abstracting the central focus of instruction, rather than pattern noticing.

The five concepts identified in this section represent only a part of the knowledge needed for teaching. They represent concepts that emerged in our work as important and needed by current teachers. They provide examples of what might be meant by key pedagogical concepts and should provoke discussion of this particular set of concepts.

An Agenda for Research on Mathematics Teacher Development

Research on mathematics teacher development can be enhanced by the articulation of clear goals for teacher learning, goals that can help to define what counts as successful learning. Teaching experiments with teachers (Simon, 2000) can be structured around a clear set of learning goals.

Although we have worked with and studied a number of fine, reform-oriented teachers, the teachers have generally not demonstrated an understanding of the concepts identified above. There is a need for research that can inform efforts to engender teacher learning with respect to these concepts. For each concept we can ask the questions:

- To what extent can teachers at different stages of professional development come to understand this concept?
- What is the process of development for each concept and how can development of the concept be fostered?
- How are concepts related in terms of prerequisite concepts and co-developing concepts?

The identification of pedagogical concepts that can serve as goals of teacher education is the first step in establishing and enacting a research agenda on mathematics teacher development.

Endnotes

1. We use "teacher education" to include both pre-service and in-service education unless otherwise specified.

2. Perhaps the most clearly articulated principled approach to mathematics instruction is Realistic Mathematics Education (RME) in the Netherlands (Gravemeijer, 1994). Its principles deal primarily with curriculum development, but they can be seen as providing a framework for mathematics teaching as well.

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